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Terms and symbols for flight dynamics — Part III : derivatives of forces, moments and their coefficients

*Termes et symboles de la mécanique du vol —
Partie III : dérivées des forces, des moments et de leurs coefficients*

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FOREWORD

ISO (the International Organization for Standardization) is a worldwide federation of national standards institutes (ISO Member Bodies). The work of developing International Standards is carried out through ISO Technical Committees. Every Member Body interested in a subject for which a Technical Committee has been set up has the right to be represented on that Committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work.

Draft International Standards adopted by the Technical Committees are circulated to the Member Bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 1151/III (previously ISO 1153) was drawn up by Technical Committee ISO/TC 20, *Aircraft and space vehicles*.

It was approved in April 1971 by the Member Bodies of the following countries :

Austria	Greece	Spain
Belgium	Israel	Thailand
Brazil	Italy	Turkey
Czechoslovakia	Japan	United Kingdom
Egypt, Arab Rep. of	Netherlands	U.S.S.R.
France	New Zealand	
Germany	South Africa, Rep. of	

No Member Body expressed disapproval of the document.

This International Standard cancels and replaces ISO Recommendation R 1153-1969.

International Standard ISO 1151/III, *Terms and symbols for flight dynamics – Part III : Derivatives of forces, moments and their coefficients*, is the third in a series of International Standards, the purpose of which is to define the principal terms used in flight dynamics and to specify symbols for these terms.

Other International Standards in this series, which will be extended in the future, are at present as follows :

ISO 1151/I, *Terms and symbols for flight dynamics – Part I : Aircraft motion relative to the air*.

ISO 1151/II, *Terms and symbols for flight dynamics – Part II : Motions of the aircraft and the atmosphere relative to the Earth*.

ISO 1151/IV, *Terms and symbols for flight dynamics – Part IV : Parameters used in the study of aircraft stability and control*.

ISO 1151/V, *Terms and symbols for flight dynamics – Part V : Quantities used in measurements*.

ISO 1151/VI, *Terms and symbols for flight dynamics – Part VI : Aircraft geometry*.

In these International Standards, the term "aircraft" denotes an aerodyne having a fore-and-aft plane of symmetry. This plane is determined by the geometrical characteristics of the aircraft. When there are more than one fore-and-aft planes of symmetry, the reference plane of symmetry is arbitrary and it is necessary to indicate the choice made.

Angles of rotation, angular velocities and moments about any axis are positive clockwise when viewed in the positive direction of the axis.

All the axis systems used are three-dimensional, orthogonal and right-handed, which implies that a clockwise (positive) rotation through $\pi/2$ about the x -axis brings the y -axis into the position previously occupied by the z -axis.

Numbering of sections and clauses

Each of these International Standards represents a part of the whole study on terms and symbols for flight dynamics.

To permit easier reference to a section or a clause from one part to another, a decimal numbering has been adopted which begins in each International Standard with the number of the part it represents.

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Terms and symbols for flight dynamics —

Part III : derivatives of forces, moments and their coefficients

3.0 INTRODUCTION

This International Standard deals with the derivatives of forces, moments and their coefficients.

In this International Standard, the effects of the Earth's curvature are not considered; for the purpose of the definition of Earth axes, the Earth's surface is treated as a plane, that is, the Earth's radius is taken as infinite.

Aeroelastic effects would introduce further quantities, which are not considered in this International Standard.

Two groups of derivatives are usually involved in flight dynamics studies.

The first group is composed of partial derivatives of the non-dimensional coefficients of force and moment components with respect to normalized variables.

The second group is composed of normalized forms of the partial derivatives of force and moment components with respect to the basic variables of the motion.

Derivatives of the first group

Aerodynamic data are usually quoted in terms of these derivatives. They are meaningful when they relate to those forces and moments which obey the laws of aerodynamic similarity but their use may be extended to include forces not of this type, for example propulsion forces.

In this International Standard, it is assumed that the force and moment coefficients depend only on the following variables¹⁾ :

- the angle of attack (1.2.2);
- the angle of sideslip (1.2.1);
- the normalized angular velocities (1.3.7);
- the Mach number (1.3.3);
- the normalized forms of the derivatives, with respect to time, of the angle of attack, the angle of sideslip, and the airspeed (3.1).

Derivatives of the second group

The derivatives of the second group relate to components of the resultant force and moment arising from all forces and moments acting on the aircraft, excluding the gravitational, inertial and reaction forces due to contact with the Earth (1.5.1).

It is usual to restrict the number of variables used in forming these derivatives to the following²⁾ (1.3.4 and 1.3.6) :

$$u, v, w, p, q, r, \dot{u}, \dot{v}, \dot{w},$$

where the dot refers to differentiation with respect to time.

These partial derivatives are then normalized by dividing by an appropriate constant quantity based on a datum flight condition (normally an equilibrium condition and denoted by a suffix e).

Remark on the written form of the derivative

The partial derivative, $\partial A / \partial \lambda$, of a quantity A , with respect to a variable λ , may be written A_λ or A^λ according to the custom of the country. The first form, A_λ , is used in this International Standard.

1) In exceptional circumstances it may prove necessary to increase the number of variables, for example to include Reynolds number.

2) In exceptional circumstances it may prove necessary to increase the number of variables, for example to include altitude.

3.1 NORMALIZED FORMS OF THE DERIVATIVES, WITH RESPECT TO TIME, OF THE ANGLES OF ATTACK AND SIDESLIP, AND AIRSPEED

No.	Term	Definition	Symbol
3.1.1	Normalized rate of change of the angle of attack	The derivative of the angle of attack (1.2.2) with respect to time multiplied by the factor l/V (1.3.1, 1.4.6) $\dot{\alpha}^* = \frac{\dot{\alpha} l}{V}$	$\dot{\alpha}^*$
3.1.2	Normalized rate of change of the angle of sideslip	The derivative of the angle of sideslip (1.2.1) with respect to time multiplied by the factor l/V (1.3.1, 1.4.6) $\dot{\beta}^* = \frac{\dot{\beta} l}{V}$	$\dot{\beta}^*$
3.1.3	Normalized tangential acceleration	The derivative of the airspeed (1.3.1) with respect to time multiplied by the factor l/V^2 (1.3.1, 1.4.6) $\dot{V}^* = \frac{\dot{V} l}{V^2}$	$\dot{V}^*$

3.2 DERIVATIVES OF THE NON-DIMENSIONAL COEFFICIENTS OF THE COMPONENTS OF THE RESULTANT FORCE AND MOMENT WITH RESPECT TO NORMALIZED QUANTITIES (FIRST GROUP)

No.	Term	Definition	Symbol
3.2.1	Derivatives with respect to angle of attack	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the angle of attack (1.2.2) $\begin{aligned} \partial C_X / \partial \alpha \\ \partial C_Y / \partial \alpha \\ \partial C_Z / \partial \alpha \\ \partial C_l / \partial \alpha \\ \partial C_m / \partial \alpha \\ \partial C_n / \partial \alpha \end{aligned}$	$\begin{aligned} C_{X\alpha} \\ C_{Y\alpha} \\ C_{Z\alpha} \\ C_{l\alpha} \\ C_{m\alpha} \\ C_{n\alpha} \end{aligned}$

No.	Term	Definition	Symbol
3.2.2	Derivatives with respect to the angle of sideslip	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the sideslip angle (1.2.1) $\partial C_X / \partial \beta$ $\partial C_Y / \partial \beta$ $\partial C_Z / \partial \beta$ $\partial C_l / \partial \beta$ $\partial C_m / \partial \beta$ $\partial C_n / \partial \beta$	$C_{X\beta}$ $C_{Y\beta}$ $C_{Z\beta}$ $C_{l\beta}$ $C_{m\beta}$ $C_{n\beta}$
3.2.3	Derivatives with respect to the normalized rate of roll	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of roll (1.3.7) $\partial C_X / \partial p^*$ $\partial C_Y / \partial p^*$ $\partial C_Z / \partial p^*$ $\partial C_l / \partial p^*$ $\partial C_m / \partial p^*$ $\partial C_n / \partial p^*$	C_{Xp} C_{Yp} C_{Zp} C_{lp} C_{mp} C_{np}
3.2.4	Derivatives with respect to the normalized rate of pitch	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of pitch (1.3.7) $\partial C_X / \partial q^*$ $\partial C_Y / \partial q^*$ $\partial C_Z / \partial q^*$ $\partial C_l / \partial q^*$ $\partial C_m / \partial q^*$ $\partial C_n / \partial q^*$	C_{Xq} C_{Yq} C_{Zq} C_{lq} C_{mq} C_{nq}

No.	Term	Definition	Symbol
3.2.5	Derivatives with respect to the normalized rate of yaw	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of yaw (1.3.7) $\partial C_X / \partial r^*$ $\partial C_Y / \partial r^*$ $\partial C_Z / \partial r^*$ $\partial C_l / \partial r^*$ $\partial C_m / \partial r^*$ $\partial C_n / \partial r^*$	C_{Xr} C_{Yr} C_{Zr} C_{lr} C_{mr} C_{nr}
3.2.6	Derivatives with respect to Mach number	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to Mach number (1.3.3) $\partial C_X / \partial M$ $\partial C_Y / \partial M$ $\partial C_Z / \partial M$ $\partial C_l / \partial M$ $\partial C_m / \partial M$ $\partial C_n / \partial M$ NOTE — M may be replaced by M_a or η .	C_{XM} C_{YM} C_{ZM} C_{lM} C_{mM} C_{nM}
3.2.7	Derivatives with respect to the normalized rate of change of the angle of attack	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of change of the angle of attack (3.1.1) $\partial C_X / \partial \dot{\alpha}^*$ $\partial C_Y / \partial \dot{\alpha}^*$ $\partial C_Z / \partial \dot{\alpha}^*$ $\partial C_l / \partial \dot{\alpha}^*$ $\partial C_m / \partial \dot{\alpha}^*$ $\partial C_n / \partial \dot{\alpha}^*$	$C_{X\dot{\alpha}}$ $C_{Y\dot{\alpha}}$ $C_{Z\dot{\alpha}}$ $C_{l\dot{\alpha}}$ $C_{m\dot{\alpha}}$ $C_{n\dot{\alpha}}$

No.	Term	Definition	Symbol
3.2.8	Derivatives with respect to the normalized rate of change of sideslip angle	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of change of the sideslip angle (3.1.2) $\partial C_X / \partial \dot{\beta}^*$ $\partial C_Y / \partial \dot{\beta}^*$ $\partial C_Z / \partial \dot{\beta}^*$ $\partial C_l / \partial \dot{\beta}^*$ $\partial C_m / \partial \dot{\beta}^*$ $\partial C_n / \partial \dot{\beta}^*$	$C_{X\dot{\beta}}$ $C_{Y\dot{\beta}}$ $C_{Z\dot{\beta}}$ $C_{l\dot{\beta}}$ $C_{m\dot{\beta}}$ $C_{n\dot{\beta}}$
3.2.9	Derivatives with respect to the normalized tangential acceleration	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized tangential acceleration (3.1.3) $\partial C_X / \partial \dot{V}^*$ $\partial C_Y / \partial \dot{V}^*$ $\partial C_Z / \partial \dot{V}^*$ $\partial C_l / \partial \dot{V}^*$ $\partial C_m / \partial \dot{V}^*$ $\partial C_n / \partial \dot{V}^*$	$C_{X\dot{V}}$ $C_{Y\dot{V}}$ $C_{Z\dot{V}}$ $C_{l\dot{V}}$ $C_{m\dot{V}}$ $C_{n\dot{V}}$

3.3 REFERENCE QUANTITIES USED TO FORM DERIVATIVES OF THE SECOND GROUP

No.	Term	Definition	Symbol
3.3.1	Datum speed	An arbitrary, constant value of the speed used in forming derivatives of the second group, usually the equilibrium airspeed	V_e
3.3.2	Datum (air) density	An arbitrary, constant value of the air density used in forming derivatives of the second group, usually that corresponding to equilibrium flight conditions	ρ_e

3.4 NORMALIZED DERIVATIVES OF THE COMPONENTS OF THE RESULTANT FORCE AND MOMENT WITH RESPECT TO THE MOTION VARIABLES (SECOND GROUP)

NOTE — In applications in which confusion may arise between the symbols for the non-dimensional derivatives and their ordinary dimensional counterparts, a suitable dressing may be added to the symbol to distinguish between these quantities.

No.	Term	Definition	Symbol
3.4.1	Aero-normalized derivatives with respect to the velocity component along the longitudinal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the aircraft velocity component u (1.3.4) in the body axis system, expressed in non-dimensional form $(\partial X/\partial u)/\frac{1}{2} \rho_e V_e S$ $(\partial Y/\partial u)/\frac{1}{2} \rho_e V_e S$ $(\partial Z/\partial u)/\frac{1}{2} \rho_e V_e S$ $(\partial L/\partial u)/\frac{1}{2} \rho_e V_e S l$ $(\partial M/\partial u)/\frac{1}{2} \rho_e V_e S l$ $(\partial N/\partial u)/\frac{1}{2} \rho_e V_e S l$	X_u Y_u Z_u L_u M_u N_u
3.4.2	Aero-normalized derivatives with respect to the velocity component along the transverse axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the aircraft velocity component v (1.3.4) in the body axis system, expressed in non-dimensional form $(\partial X/\partial v)/\frac{1}{2} \rho_e V_e S$ $(\partial Y/\partial v)/\frac{1}{2} \rho_e V_e S$ $(\partial Z/\partial v)/\frac{1}{2} \rho_e V_e S$ $(\partial L/\partial v)/\frac{1}{2} \rho_e V_e S l$ $(\partial M/\partial v)/\frac{1}{2} \rho_e V_e S l$ $(\partial N/\partial v)/\frac{1}{2} \rho_e V_e S l$	X_v Y_v Z_v L_v M_v N_v
3.4.3	Aero-normalized derivatives with respect to the velocity component along the normal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the aircraft velocity component w (1.3.4) in the body axis system, expressed in non-dimensional form $(\partial X/\partial w)/\frac{1}{2} \rho_e V_e S$ $(\partial Y/\partial w)/\frac{1}{2} \rho_e V_e S$ $(\partial Z/\partial w)/\frac{1}{2} \rho_e V_e S$ $(\partial L/\partial w)/\frac{1}{2} \rho_e V_e S l$ $(\partial M/\partial w)/\frac{1}{2} \rho_e V_e S l$ $(\partial N/\partial w)/\frac{1}{2} \rho_e V_e S l$	X_w Y_w Z_w L_w M_w N_w

No.	Term	Definition	Symbol
3.4.4	Aero-normalized derivatives with respect to the rate of roll (1.3.6)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the rate of roll (1.3.6), expressed in non-dimensional form $(\partial X/\partial p)/\frac{1}{2} \rho_e V_e S l$ $(\partial Y/\partial p)/\frac{1}{2} \rho_e V_e S l$ $(\partial Z/\partial p)/\frac{1}{2} \rho_e V_e S l$ $(\partial L/\partial p)/\frac{1}{2} \rho_e V_e S l^2$ $(\partial M/\partial p)/\frac{1}{2} \rho_e V_e S l^2$ $(\partial N/\partial p)/\frac{1}{2} \rho_e V_e S l^2$	X_p Y_p Z_p L_p M_p N_p
3.4.5	Aero-normalized derivatives with respect to the rate of pitch (1.3.6)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the rate of pitch (1.3.6), expressed in non-dimensional form $(\partial X/\partial q)/\frac{1}{2} \rho_e V_e S l$ $(\partial Y/\partial q)/\frac{1}{2} \rho_e V_e S l$ $(\partial Z/\partial q)/\frac{1}{2} \rho_e V_e S l$ $(\partial L/\partial q)/\frac{1}{2} \rho_e V_e S l^2$ $(\partial M/\partial q)/\frac{1}{2} \rho_e V_e S l^2$ $(\partial N/\partial q)/\frac{1}{2} \rho_e V_e S l^2$	X_q Y_q Z_q L_q M_q N_q
3.4.6	Aero-normalized derivatives with respect to the rate of yaw (1.3.6)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the rate of yaw (1.3.6), expressed in non-dimensional form $(\partial X/\partial r)/\frac{1}{2} \rho_e V_e S l$ $(\partial Y/\partial r)/\frac{1}{2} \rho_e V_e S l$ $(\partial Z/\partial r)/\frac{1}{2} \rho_e V_e S l$ $(\partial L/\partial r)/\frac{1}{2} \rho_e V_e S l^2$ $(\partial M/\partial r)/\frac{1}{2} \rho_e V_e S l^2$ $(\partial N/\partial r)/\frac{1}{2} \rho_e V_e S l^2$	X_r Y_r Z_r L_r M_r N_r

No.	Term	Definition	Symbol
3.4.7	Aero-normalized derivatives with respect to the rate of change of the velocity component along the longitudinal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to $\dot{u}$ (see Introduction), expressed in non-dimensional form $(\partial X / \partial \dot{u}) / \frac{1}{2} \rho_e S l$ $(\partial Y / \partial \dot{u}) / \frac{1}{2} \rho_e S l$ $(\partial Z / \partial \dot{u}) / \frac{1}{2} \rho_e S l$ $(\partial L / \partial \dot{u}) / \frac{1}{2} \rho_e S l^2$ $(\partial M / \partial \dot{u}) / \frac{1}{2} \rho_e S l^2$ $(\partial N / \partial \dot{u}) / \frac{1}{2} \rho_e S l^2$	$X_{\dot{u}}$ $Y_{\dot{u}}$ $Z_{\dot{u}}$ $L_{\dot{u}}$ $M_{\dot{u}}$ $N_{\dot{u}}$
3.4.8	Aero-normalized derivatives with respect to the rate of change of the velocity component along the transverse axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to $\dot{v}$ (see Introduction), expressed in non-dimensional form $(\partial X / \partial \dot{v}) / \frac{1}{2} \rho_e S l$ $(\partial Y / \partial \dot{v}) / \frac{1}{2} \rho_e S l$ $(\partial Z / \partial \dot{v}) / \frac{1}{2} \rho_e S l$ $(\partial L / \partial \dot{v}) / \frac{1}{2} \rho_e S l^2$ $(\partial M / \partial \dot{v}) / \frac{1}{2} \rho_e S l^2$ $(\partial N / \partial \dot{v}) / \frac{1}{2} \rho_e S l^2$	$X_{\dot{v}}$ $Y_{\dot{v}}$ $Z_{\dot{v}}$ $L_{\dot{v}}$ $M_{\dot{v}}$ $N_{\dot{v}}$
3.4.9	Aero-normalized derivatives with respect to the rate of change of the velocity component along the normal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to $\dot{w}$ (see Introduction), expressed in non-dimensional form $(\partial X / \partial \dot{w}) / \frac{1}{2} \rho_e S l$ $(\partial Y / \partial \dot{w}) / \frac{1}{2} \rho_e S l$ $(\partial Z / \partial \dot{w}) / \frac{1}{2} \rho_e S l$ $(\partial L / \partial \dot{w}) / \frac{1}{2} \rho_e S l^2$ $(\partial M / \partial \dot{w}) / \frac{1}{2} \rho_e S l^2$ $(\partial N / \partial \dot{w}) / \frac{1}{2} \rho_e S l^2$	$X_{\dot{w}}$ $Y_{\dot{w}}$ $Z_{\dot{w}}$ $L_{\dot{w}}$ $M_{\dot{w}}$ $N_{\dot{w}}$