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TECHNICAL REPORT

colour inside

Guidelines for the statistical analysis of ageing test data – Part 3: Minimum specimen numbers at different test conditions with given experimental data

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GUIDELINES FOR THE STATISTICAL ANALYSIS OF AGEING TEST DATA –

Part 3: Minimum specimen numbers at different test conditions with given experimental data

FOREWORD

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IEC TR 60493-3, which is a Technical Report, has been prepared by IEC technical committee 112: Evaluation and qualification of electrical insulating materials and systems.

The text of this Technical Report is based on the following documents:

Enquiry draft	Report on voting
112/384/DTR	112/389/RVDTR

Full information on the voting for the approval of this Technical Report can be found in the report on voting indicated in the above table.

This document has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 60493 series, published under the general title *Guidelines for the statistical analysis of ageing test data*, can be found on the IEC website.

Future standards in this series will carry the new general title as cited above. Titles of existing standards in this series will be updated at the time of the next edition.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific document. At this date, the document will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

IMPORTANT – The 'colour inside' logo on the cover page of this publication indicates that it contains colours which are considered to be useful for the correct understanding of its contents. Users should therefore print this document using a colour printer.

INTRODUCTION

The objective of this document is to clarify how a statistical analysis can be done even with a small number of samples.

When the scatter of data is sufficiently small, a statistical analysis should be possible. Generally, a statistical analysis applies to a larger number of samples similar to ordinary cases.

On the other hand, this document may be useful in clarifying how the scatter of a small number of data points can be used to estimate "lifetime" when the number of specimens is limited (e.g. around five) and there are few ageing conditions (e.g. two or three conditions).

Therefore, the aim is to document, for a small group of specimens with limited scatter of data, whether or not it is possible to estimate characteristics such as lifetime within a certain statistical error.

If such a simulation were available, it would be very helpful for users.

Manufacturers, for example, may prefer a new simplified, accelerated test method as an alternative to their current traditional test methods. A new test method would be easier and less expensive, especially if the specimens were small in size (dimension), although it may not be as suitable for large specimen sizes or expensive materials or devices (such as motors).

It would be very welcome if a statistical treatment method of this type, applying to a smaller number of specimens, were defined.

Though inappropriate cases are anticipated, this document deals with cases where such statistical treatment is appropriate.

GUIDELINES FOR THE STATISTICAL ANALYSIS OF AGEING TEST DATA –

Part 3: Minimum specimen numbers at different test conditions with given experimental data

1 Scope

This part of IEC 60493 clarifies how a statistical analysis can be done with a small number of samples.

This document will be useful when the accelerated test method is difficult to carry out, for example in cases where the dimensions of test specimens (including test devices) are very large in scale or the cost of test specimens is high. Testing is facilitated by enabling users to reduce the number of test specimens.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60493-1, *Guide for the statistical analysis of ageing test data – Part 1: Methods based on mean values of normally distributed test results*

IEC TR 60493-2, *Guide for the statistical analysis of ageing test data – Part 2: Validation of procedures for statistical analysis of censored normally distributed data*

IEC 62539, *Guide for the statistical analysis of electrical insulation breakdown data*

3 Terms, definitions, symbols and conventions

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60493-1, IEC TR 60493-2, and IEC 62539 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at http://www.electropedia.org/
- ISO Online browsing platform: available at http://www.iso.org/obp

3.2 Symbols

In this document, the following symbols are used.

Table 1 – Symbols

A	pre-factor in equation to express rate constant
A^*, c/A	pre-factor in equation to express lifetime
c	criterion of degradation
k	rate constant of degradation
μ	mean of lifetime
R	ideal gas constant (= 8,31 J $K^{-1}mol^{-1}$)
σ	standard deviation
σ_i	standard deviation of lifetime at temperature i
σ_H	standard deviation of lifetime at high temperature
σ_M	standard deviation of lifetime at medium temperature
σ_L	standard deviation of lifetime at low temperature
T	absolute temperature, in K
t	time
t_H	lifetime at high temperature
t_i	lifetime at temperature i
t_M	lifetime at medium temperature
t_L	lifetime at low temperature
τ	lifetime (time to criterion)
U	activation energy of a chemical reaction, in eV or J mol^{-1}

3.3 Conventions

In this document, τ (Greek tau) means lifetime in use, t (lower case) means lifetime upon accelerated ageing and T (upper case) means temperature. Subscripts H, M and L mean high, medium and low temperatures, respectively (see Table 1).

- Lifetime τ_u at temperature T_u is to be estimated.
- The scatter of τ_u shall be below a criterion, where the criterion is decided by the user.
- Through an accelerated test, τ_u can be derived with t_H at T_H, t_M at T_M and t_L at T_L, where $T_H > T_M > T_L > T_u$.
- t_H, t_M and t_L have their own scatter. Generally, they may differ from each other. For convenience, scatter shall be assumed to be equal to the first step.
- This document aims to clarify how small the scatter of t_H, t_M and t_L needs to be in order to keep the scatter of τ_u below the criterion.
- This document also aims to clarify how large the number of test specimens needs to be in order to achieve a small scatter of t_H, t_M and t_L, depending on the intrinsic scatter of the specimens.

4 Assumptions

a) The type of ageing (stressor) determines which physical or chemical laws are obeyed. As a first step, thermal ageing shall be considered; other stressors shall be discussed in other documents. The Arrhenius law is applied. The rate constant of degradation (for example, chemical degradation), k, is written as follows, where A is a pre-factor, U is the activation energy (typically in J mol^{-1}), R is the gas constant (8,31 J $K^{-1}mol^{-1}$) and T is the absolute temperature in Kelvin (K). Accordingly, the time to a specific degradation or failure (at a fixed ageing temperature T), referred to here as the lifetime, is given as follows, where c is a constant (not dimensionless, criterion of degradation) and $A^* = c/A$;

$$k = A\exp\left[-\frac{U}{\mathrm{R}T}\right],\ \tau = (\tau_T) = (c/A)\exp\left[+\frac{U}{\mathrm{R}T}\right] = A^*\exp\left[+\frac{U}{\mathrm{R}T}\right],$$

b) Lifetime is assumed to obey the normal distribution, with the parameters of standard deviation σ and mean μ.

$$f(t;T) = \frac{1}{\sqrt{2\pi}\,\sigma}exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right],\ \mu = \tau = A*\exp\left[+\frac{U}{\mathrm{R}T}\right]$$

This document considers the case where lifetime follows a normal distribution. In other cases, lifetime may follow other distributions, considered in other documents.

c) This document considers the survival ratio, i.e., (1-N_{failure}/N_0 = N_{survive}/N_0), not the ratio of a residual property to its initial value (p/p_0, where p is a property such as elongation at break, tensile or flexural strength, modulus) as a function of ageing time. The latter shall be described in future document(s) of the IEC 60493 series.

d) The activation energy U and pre-factor A^* shall be assumed constants independent of temperature. In other words, this document considers a temperature range wherein constant E and A^* can be assumed.

5 General considerations

A case shall be considered where the number of specimens is n = 5 per temperature with three temperatures.

The ageing temperatures shall be T_{H}, T_{M}, and T_{L} (subscripts indicating high, medium and low). The actual or practical temperature of use is written as T_{u} (where $T_{\mathrm{H}} > T_{\mathrm{M}} > T_{\mathrm{L}} > T_{\mathrm{u}}$). The decrement in temperature should be at most 10 °C to 20 °C. It shall be assumed that within this temperature range, there are no physical or chemical transitions, nor changes in degradation scheme.

The time to failure at the ageing temperature of T_{H} for five specimens shall be on the order of $t_{1\mathrm{H}}$, $t_{2\mathrm{H}}$, $t_{3\mathrm{H}}$, $t_{4\mathrm{H}}$, $t_{5\mathrm{H}}$ (from short to long). Similar survival curves can be drawn for the ageing temperatures of T_{M} and T_{L} (see Figure 1).

From the test results, $\bar{t}_{\mathrm{H}}$ (mean time to failure), σ_{H} (standard deviation of time to failure) can be derived as follows (where σ shall be unbiased).

$$\bar{t}_{\mathrm{H}} = \frac{\sum t_{i\mathrm{H}}}{n},\ \sigma_{\mathrm{H}} = \frac{\sum\left(\bar{t}_{\mathrm{H}} - t_{i\mathrm{H}}\right)^2}{n-1} \quad \text{(in this example, } n = 5\text{)}$$

A similar calculation is done for the ageing temperatures of T_{M} and T_{L}.

It is expected that $\bar{t}_{\mathrm{H}} < \bar{t}_{\mathrm{M}} < \bar{t}_{\mathrm{L}}$. In other words, the higher the ageing temperature, the faster degradation occurs (and vice versa; namely, the lower the ageing temperature, the slower degradation occurs), a phenomenon which is well accepted. If this is not the case, the tests may be repeated.

It is expected that $\sigma_{\mathrm{H}} < \sigma_{\mathrm{M}} < \sigma_{\mathrm{L}}$. In other words, the higher the ageing temperature, the smaller the scatter due to fast degradation (and vice versa; namely, the lower the ageing temperature, the larger the scatter). However, this assumption is controversial and may not always hold.

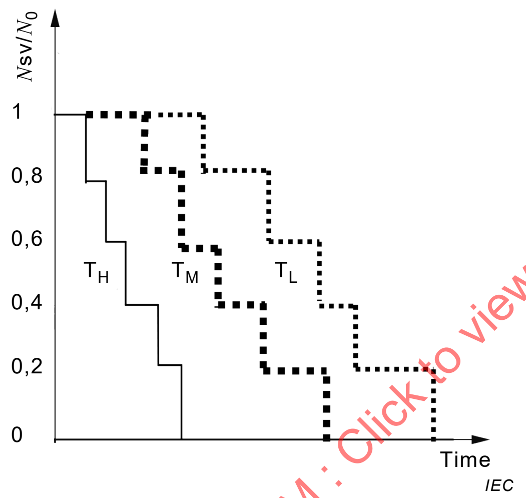
The estimated failure time $\bar{t}_{\mathrm{u}}$ and scatter (standard deviation) σ_{u} can be derived from an Arrhenius plot (see Figure 2).

Make the plot of $1/T$ versus $\frac{\sigma}{t}$.

The series of $\frac{\sigma_i}{t_i}$ values should be constant, independent of ageing temperature, and below a certain criterion. This criterion can be determined by users.

If a certain tendency is observed between the series of $\frac{\sigma_i}{t_i}$ values and the ageing temperature, then the σ/t value at T_u, namely, $\frac{\sigma_u}{t_u}$ should be calculated by using some approximation such as the least square for $1/T$ versus $\frac{\sigma}{t}$. This expected $\frac{\sigma_u}{t_u}$ should be below a certain criterion. This criterion can be determined by users (see Figure 3).

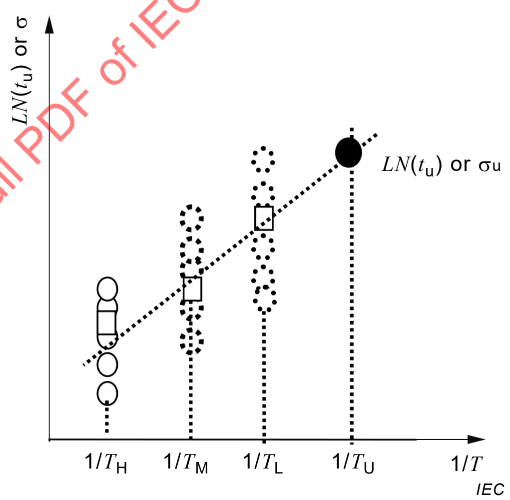
If the above conditions are not satisfied, increasing the number of specimens should be considered which would then result in the ordinary case.



Key

In the case of n = 5 for each temperature and 3 temperatures

Figure 1 – Ageing time versus survival ratio



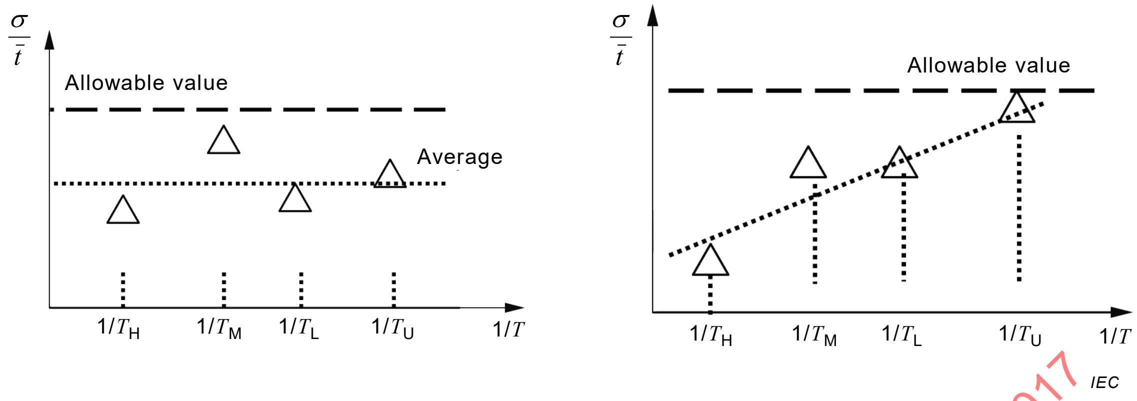
Key

Open circle : Data point for failure time

Open square : Average or standard deviation

Solid circle : Estimate

Figure 2 – Arrhenius plot of failure time or scatter



Key

Left: Constant $\frac{\sigma}{\bar{t}}$;

Right: $\frac{\sigma}{\bar{t}}$ has some tendency;

Bashed line: Allowable value

Figure 3 – Temperature dependence of $\frac{\sigma}{\bar{t}}$

6 Probability calculation based on normal distributions

6.1 General

Clause 6 provides an example of how the standard deviation of samples and the number of samples (sample size) affect the estimation of lifetime.

6.2 Calculations

Here the basic concept is explained of how simulation results may change due to the number of data points and data scatter. Even if the number of data points is small, the distribution may give a sufficiently reliable estimate of the simulated life or simulated strength of the target object if the scatter of data is sufficiently small. Based on this idea, the distribution patterns are analysed under different statistical situations using the normal distribution as the first trial. Probability calculations have been done with normal distributions having different standard deviations and mean values to simulate the number of data points and data scatter using the following equations.

$$\text{Probability}(t)=\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(t-t_\mathrm{M})^2}{2\sigma^2}\right],\ \text{where}\ t_\mathrm{M}=A^*\exp\left(+\frac{U}{\mathrm{R}T}\right)$$

In this example, the activation energy (U) is set to be 2 eV (= 192 kJ mol^{-1} ≈ 46 kcal mol^{-1}) with different standard deviations for each life estimation case. Life test temperatures are set to be 170 °C, 150 °C and 130 °C and the life at 90 °C and 50 °C are estimated with several numbers of data points and scatter values. N_d in all the figures is the number of data points at each test temperature and σ is the standard deviation that may be changed to 0,05, 0,1, 0,2 and 0,3. N_s stands for the number of the data-sets of the simulation for each temperature.

In the following Subclauses 6.3 and 6.4, N_s is set to be 1 000. In several figures such as Figures 5, 6, 8 and 9, there is a label "$N_\mathrm{c}/N_\mathrm{s}$" which designates the effective ratio indicating the number of data points for each case, i.e., 170 °C, 150 °C, 130 °C and the target temperature. Basically $N_\mathrm{c}/N_\mathrm{s}$ corresponds to the displayed data for each temperature.

Random numbers are used to simulate the life data. At each temperature the number of data items is fixed and data is generated up to that number, such as 20. Once the random number data x ($0 < x < 1$) is generated, then x is changed to life data at a given temperature using the following equation.

$$\text{Lifetime} = \text{erfinv}(2x-1)\ ,\ \text{i.e.,}\ \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\text{Lifetime}} \exp\left[-\frac{(t-t_{\text{M}})^2}{2\sigma^2}\right]dt = x$$

Here erfinv (y) is an inverse error function given by MATLAB®[1]. Basically, all calculations and data graphing are done with MATLAB® software.

Using the procedure above, some randomly-generated x can result in a negative lifetime (in the case where a small value of x is generated), and that is physically meaningless. In the case studied, those values were omitted. Thus the effective number of data points is equal to or less than the number of trials, which is the reason to consider N_{c}.

Notations:

N_{d}: is the number of data points, typically 3, 5, 10, 20, etc.

$N_{\text{d}i}$: is the number of data points in the i^{th} simulation, where $i = 1, .., N_{\text{s}}$.

N_{s}: is the number of simulations, in 6.3 and 6.4, N_{s} = 1 000.

N_{c}: $\sum_i N_{\text{d}i}$ $(i = 1, .., N_{\text{s}})$, where $\sum_i N_{\text{d}i}$ can be less than $N_{\text{d}} {}^* N_{\text{s}}$; if a negative lifetime is generated, it is not used.

Simulation plots are made as follows:

a) Fix the temperature T.
b) Generate x ($0 < x < 1$) randomly.
c) Based on x (and T), a lifetime is given.
d) Repeat steps b) and c) N_{d} times.
e) Change the temperature and repeat the steps from b) to d) (for all three temperatures).
f) Draw a regression line by using the simulation data obtained above.
g) Estimate lifetime τ_{u} at the temperature of use, T_{u}.
h) Repeat the steps from a) to g) by N_{s} (1 000) times.
i) Thus, N_{s} (1 000) plots can be overlaid along the cumulative probability curve.

The following Subclauses 6.4 and 6.5 demonstrate the figures indicating the theoretical cumulative probability curves and simulation plots with the value of A^* appropriately chosen.

The general tendencies observed from the figures in 6.3 and 6.4, as conclusions derived from this simulation, can be summarized as follows:

The greater N_{d}, the smaller the deviation (and vice versa), which is reasonable.

N_{d}	few	→	many
Deviation	large	→	small

1 MATLAB® is the trademark of a product supplied by MathWorks. This information is given for the convenience of users of this document and does not constitute an endorsement by IEC of the product named. Equivalent products may be used if they can be shown to lead to the same results.

a) The higher σ, the larger the deviation.

σ	low	→	high
Deviation	small	→	large

b) The higher ΔT, the larger the deviation.

ΔT	low	→	high
Deviation	small	→	large

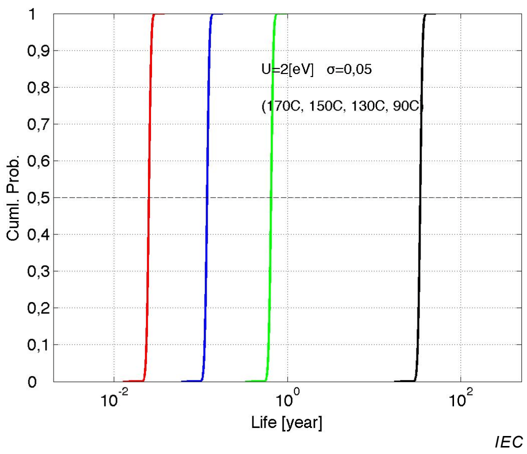
c) The greater N_d and the lower σ, the smaller the deviation.

N_d \ σ	low	→	high
many	small		
↓		deviation	
few			large

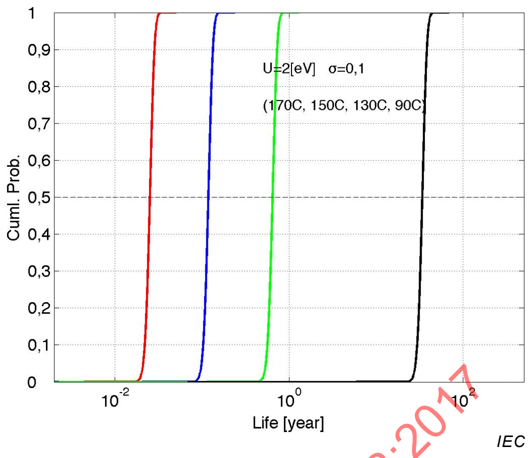
6.3 Estimated life at 90 °C (temperature difference is set to be 40 °C)

6.3.1 Ideal case with σ = 0,05, 0,1, 0,2, 0,3

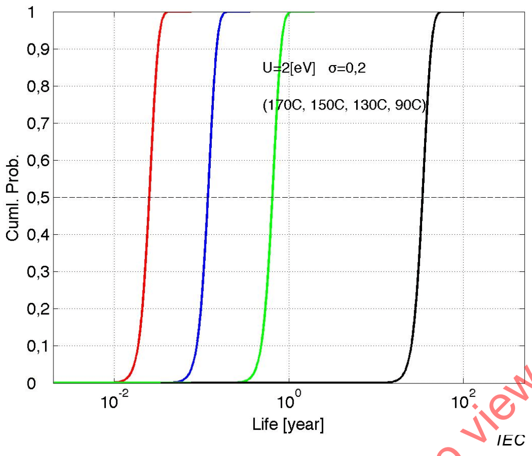
Figure 4 shows curves of the cumulative probability as a function of normal logarithmic lifetime for various standard deviations from 0,05 to 0,3 for the ideal case; where "(170C, 150C, 130C, 90C)" corresponds to "(170 °C, 150 °C, 130 °C, 90 °C)", and the vertical axis "Cumul. Prob." means "Cumulative probability"; the same applies from Figure 4 to Figure 9. These are theoretical cases, and therefore the simple curves are shifted by the lifetime given by the Arrhenius equation. As can be seen, the higher the standard deviation, the more gradual the curve is, and vice versa. These curves can serve as the control for simulations.



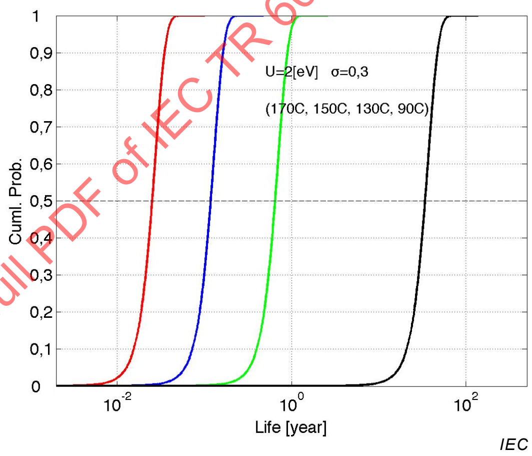
(a) σ = 0,05



(b) σ = 0,1



(c) σ = 0,2



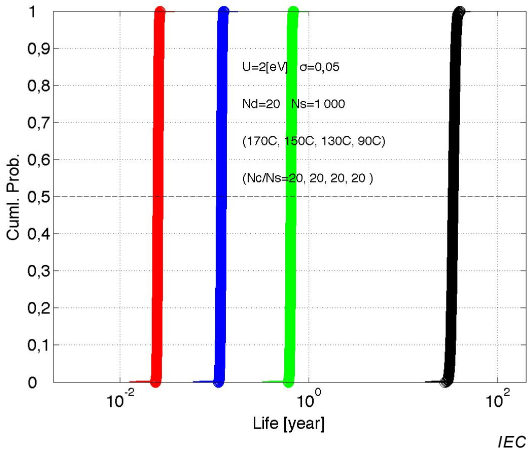
(d) σ = 0,3

Figure 4 – Estimated life at 90 °C for the ideal case

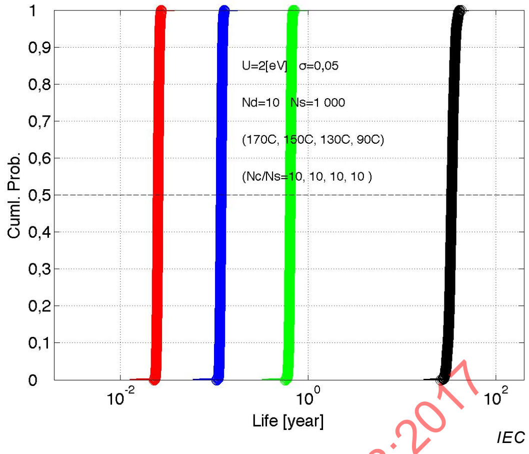
6.3.2 Test sample numbers of 20, 10, 5, 3 with σ = 0,05

Figure 5 shows curves of the cumulative probability as a function of normal logarithmic lifetime for a standard deviation of 0,05 and simulation plots are overlaid for cases whose numbers of test samples are 20, 10, 5 and 3 (denoted as N_d).

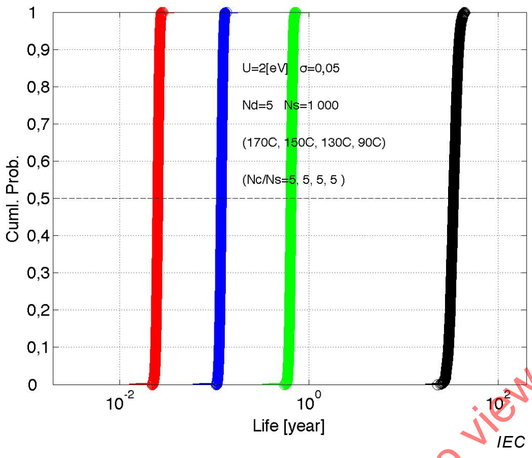
The plots match well with the theoretical curve, thus indicating that the standard deviation of 0,05 may be sufficiently small, independent of the number of data points (N_d).



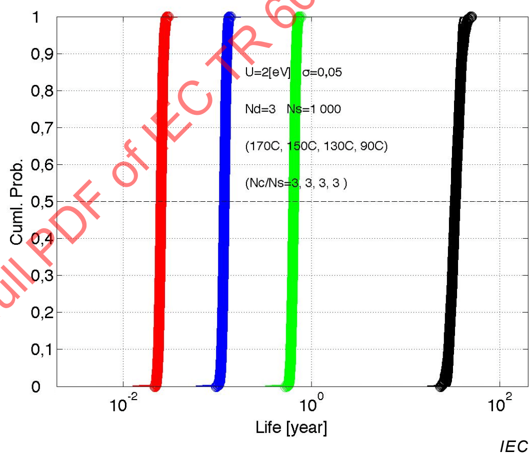
(a) σ = 0,05, N_d = 20



(b) σ = 0,05, N_d = 10



(c) σ = 0,05, N_d = 5



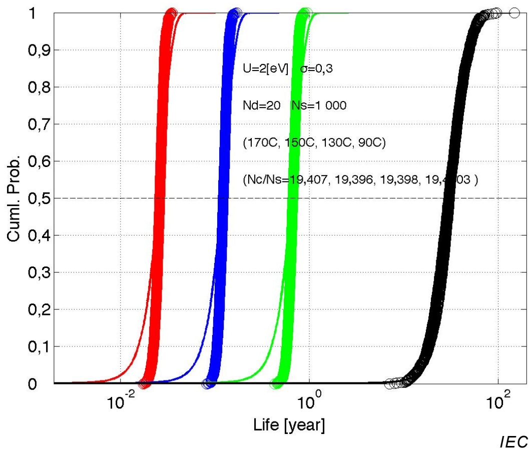
(d) σ = 0,05, N_d = 3

Figure 5 – Test sample numbers of 20, 10, 5, 3 with σ = 0,05

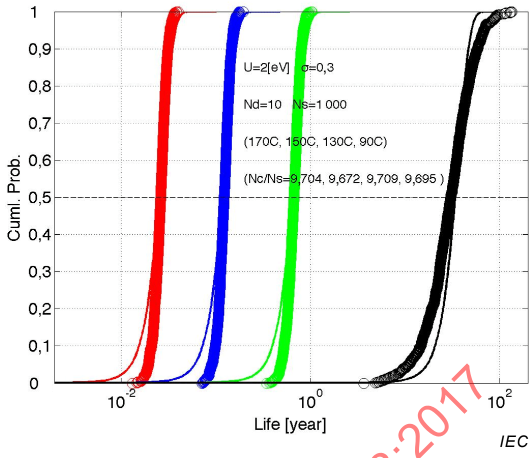
6.3.3 Test sample numbers of 20, 10, 5, 3 with σ = 0,3

Figure 6 shows curves of the cumulative probability as a function of normal logarithmic lifetime for a standard deviation of 0,3 and simulation plots are overlaid for cases whose number of test samples are 20, 10, 5 and 3 (denoted as N_d).

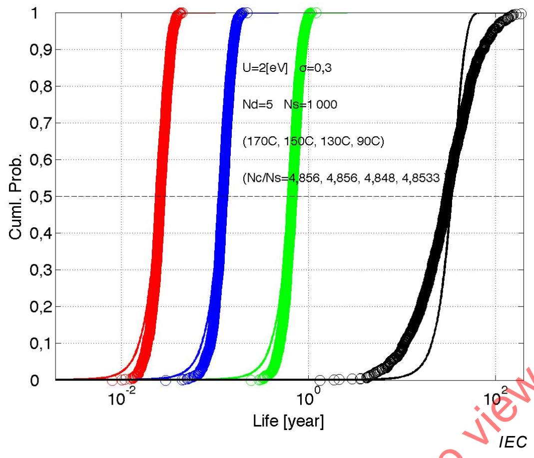
Unlike the previous case (6.3.2), some acceleration plots (shown in red, blue or green) deviate from the theoretical curve, especially in the short-time region. Afterwards, the acceleration plots agree well with the theoretical curve. However, the estimated plots (shown in black) deviate from the theoretical curve. While the difference is small for an N_d of 20, it becomes substantial with a decreasing N_d.



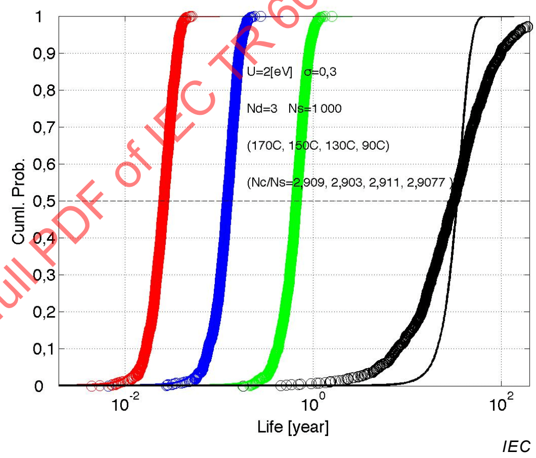
(a) σ = 0,3, N_d = 20



(b) σ = 0,3, N_d = 10



(c) σ = 0,3, N_d = 5



(d) σ = 0,3, N_d = 3

Figure 6 – Test sample numbers of 20, 10, 5, 3 with σ = 0,3

6.4 Estimated life at 50 °C (temperature difference is set to be 80 °C)

6.4.1 Ideal case with σ = 0,05, 0,1, 0,2, 0,3

Similarly to the case in 6.3.1, Figure 7 shows curves of the cumulative probability as a function of normal logarithmic lifetime for various standard deviations from 0,05 to 0,3 for the ideal case. These are the theoretical cases, and therefore the simple curves are shifted by the lifetime given by the Arrhenius equation. The difference from 6.3 is that the temperature difference between the accelerating (elevated) temperature and the temperature of use is larger; in this case it is 80 °C, while in the former it is 40 °C. As can be seen, the higher the standard deviation becomes, the more gradual the curve is, and vice versa. These curves can serve as the control for further simulations.